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LETTER TO THE EDITOR

Weak versus strong uniqueness of Gibbs measures: a regular short-range example

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Abstract. We provide an example of a nearest-neighbour random model on a regular lattice which, almost certainly, has a unique (disordered) Gibbs state for every boundary condition, although by choosing interaction-dependent boundary conditions, one can obtain different Gibbs states.

The correct treatment and interpretation of boundary conditions for statistical-mechanical systems is a subtle issue, especially for random models [1–14]. For instance, the notion of a plurality of pure states with an ultrametric structure, as in Parisi's proposal, as well as the mechanism by which they can be obtained, requires a careful treatment of boundary conditions. This issue is of great interest in spin-glass theory [15]. Another example is the distinction between the 'weak' and 'strong' uniqueness of Gibbs measures for random spin systems. We say that strong uniqueness applies to a model if, for almost all choices of the interaction, only one Gibbs measure exists, while weak uniqueness holds if, for any boundary condition chosen independently of the interaction, the thermodynamic limit of the Gibbs measure is the same for almost all interactions. Weak uniqueness was first explicitly introduced and discussed in [1] where it was proved to hold for one-dimensional Ising models with random interactions decaying as $1/r^{\alpha}$ with $\alpha > 1$; strong uniqueness is known to hold only for $\alpha > \frac{3}{2}$ [16]. The distinction between interaction-dependent as opposed to interaction-independent boundary conditions was previously discussed in [4, 5, 17] in a more implicit way.

Up till now there have been examples of systems which are weakly, but not strongly, unique for spin-glass models on the Bethe lattice [17, 18] in the temperature range between the ferromagnetic and the spin-glass transition temperatures and for extreme-long-range (square summable, but non-summable) spin-glass models [9, 19] at high temperatures. The usual interpretation of these results is that, in these examples, interaction-dependent boundary conditions should be dismissed as 'unphysical'.

Here we present for the first time a nearest-neighbour example on a regular lattice of this phenomenon. The interpretation of interaction-dependent boundary conditions as 'unphysical' seems somewhat tenuous in this case.

The model we consider is the q-state nearest-neighbour Potts model on $\mathbb{Z}^d$ at T_c for high q (and $d \geqslant 2$) with a one-pattern Hopfield-type site disorder

$$H = -\sum_{\langle i,j \rangle} \delta(\xi_i \sigma_i, \xi_j \sigma_j) \tag{1}$$

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where $\sigma_i \in \{1, \ldots, q\}$ and ξ_i is a random permutation of the q Potts states at site i [20, 21].

After a gauge transformation, the model is equivalent to a Potts ferromagnet, for which it is well known [22, 23] that at T_c there is a first-order phase transition in the temperature variable. At T_c, q ordered states (Gibbs measures) coexist with a disordered state. Any fixed boundary condition, due to the random gauge transformation, is equivalent to taking a random choice for the boundary condition for the ferromagnet. Thus, it is sufficient to argue that a sequence of finite-volume Gibbs measures for the ferromagnetic model with randomly chosen boundary conditions converges to the disordered state. To achieve this, we invoke the proof of the first-order transition due to Bricmont–Kuroda–Lebowitz [23]. This proof is based on Pirogov–Sinai theory, i.e. a sophisticated contour argument. The disordered state is a small perturbation of the 'restricted ensemble' of configurations in which all neighbouring spins are different (small for sufficiently high q). We observe that the density of pairs of neighbouring spins on the boundary which are equal is close to $1/q$ with large probability, and they are 'sparse' if the spins on the boundary are independently chosen on each boundary site, with probability $1/q$ for each of the q Potts states. This means that the boundary condition is a small random perturbation of a 'purely restricted ensemble' boundary condition. Therefore, a probabilistic contour argument, as in [24], will lead to the desired result that, with respect to the random boundary conditions, the thermodynamic limit measure will almost surely be the disordered state.

As previously mentioned, this implies the weak uniqueness of the gauge-transformed random Potts–Hopfield model.

Intuitively, it is of course plausible that uncorrelated random boundary conditions are the most 'disordered'. We note that for any extremal Gibbs measure, it is true that almost all (with respect to this same Gibbs measure) boundary conditions recover the original measure [25]. We have considered here almost all boundary conditions with respect to the symmetric product measure which, in some sense, is the most random prescription (for example, this measure has the largest entropy).

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References

[1] Campanino M, Olivieri E and van Enter A C D 1987 *Commun. Math.* **108** 241–55
[2] van Enter A C D 1990 *J. Stat. Phys.* **60** 275
[3] van Enter A C D 1988 *J. Phys. A: Math. Gen.* **21** 1781
[4] van Enter A C D and Fröhlich J 1985 *Commun. Math. Phys.* **98** 425
[5] van Enter A C D and Griffiths R B 1983 *Commun. Math. Phys.* **90** 319
[6] Fisher D S and Huse D A 1986 *Phys. Rev. Lett.* **56** 1601
[7] Fisher D S and Huse D A 1987 *J. Phys. A: Math. Gen.* **20** L1005
[8] Fisher D S and Huse D A 1988 *Phys. Rev.* B **38** 386
[9] Gandolfi A, Newman C M and Stein D L 1993 *Commun. Math. Phys.* **157** 371
[10] Huse D A and Fisher D S 1987 *J. Phys. A: Math. Gen.* **20** L997
[11] Newman C M 1994 *Probability and Phase transition (NATO ASI 420)* ed G Grimmett (Dordrecht: Kluwer) p 247
[12] Newman C M and Stein D L 1992 *Phys. Rev.* B **46** 973
[13] Ozeki 1993 *J. Phys. Soc. Japan* **62** 2641
[14] Zegarlinski B 1994 *Rev. Math. Phys.* **6** 115
[15] Mézard M, Parisi G and Virasoro M A 1987 *Spin Glass Theory and Beyond* (Singapore: World Scientific)
[16] Khanin K M 1980 *Theor. Math. Phys.* **15** 445

[17] Chayes J T, Chayes L, Sethna J P and Thouless D J 1986 *Commun. Math. Phys.* **106** 41
[18] Blekher P M, Ruiz J and Zagrebnov V A 1994 *Marseille Preprint* Austin Mathematical Physics Archives 94-296
[19] Fröhlich J and Zegarlinski B 1987 *Commun. Math. Phys.* **110** 121
[20] Kanter I 1988 *Phys. Rev.* A **37** 2739
[21] Gayrard V 1992 *J. Stat. Phys.* **68** 977
[22] Kotecký R and Shlosman S B 1982 *Commun. Math. Phys.* **83** 493
[23] Bricmont J, Kuroda K and Lebowitz J L 1985 *Commun. Math. Phys.* **101** 501
[24] Chayes J T, Chayes L and Fröhlich J 1985 *Commun. Math. Phys.* **100** 399
[25] Georgii H-O 1973 *Commun. Math. Phys.* **32** 107